Experimental objections to special relativity and first reactions (1905-1908)

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Abstract: Kaufmann experiments (1901-1905) concerning electron's charge-mass ratio resulted in an experimental objection to special relativity, in front of classic electron theories, leading to a debate within the scientific community that involved renowned physicists such as Planck and Lorentz.

I. INTRODUCTION

This project aims to explain and comprehend a particular obstacle that Einstein's special relativity had to overcome in order to be established as the predominant theory in the physics community of the 20th century. That is, Walter Kaufmann (1871-1947) experiments between 1901 and 1905 concerning electron's charge-mass variation with high velocities. These experiments compared different electron theories, classic and relativistic, and the results gave rise to an important discussion because of the criticism towards special relativity's validity.

The research methodology has been essentially bibliographic: analysing Kaufmann's results, reading original articles from the main characters of this historic events ([1], [2], [3], [4]), as well as historical books and papers ([5], [6]). With all the information collected from these different sources, it was easier to get a general perspective of the situation, and a thorough analysis was made to elaborate a scientific historical thread that summarizes the respective events held during the first decades of last century. This academic work led to the conclusions concerning Kaufmann's results and its consequences within physics and relativity. The body of this project is constructed around the explanation of Kaufmann's experiments and analysis of their results, as well as the discussion over them involving renowned physicists.

So, the main goal is to comprehend this experimental objection to special relativity, to see whether Kaufmann's results had a real historical impact or not. In the same line, it is interesting to examine the different reactions and scientific interpretations of the different physicists involved in the subsequent debate.

II. HISTORICAL CONTEXT

It is well known that science cannot be comprehended in its totality without a look back to its origins and historical development, and physics is not an exception to this rule. One of the breakthrough moments in the history of physics is the publication of the special theory of relativity in 1905 by Albert Einstein (1879-1955) [1], which revolutionized the scientific scheme of the moment, finally ending the dominance of the theory of the ether.

However, it is necessary to retrocede some years to gain a better perspective and understanding of the prevalent vision of the nature and the universe in the beginning of the 20th century: the electromagnetic world-picture. This is mostly due to James Clerk Maxwell (1831-1879), who in 1865 identified light as an electromagnetic wave, therefore unifying optics with electromagnetism.

Afterwards, electromagnetism became prevalent in physics and many scientists aimed to explain nature only by terms of this theory: mass, forces, etc. One of them was Max Abraham (1875-1922), whose theory of the electron (1902) [5, pp. 55-60] was based in a rigid sphere with an electromagnetic mass (originated by its own electromagnetic fields). As a result, the electron's inertia could be explained by the interaction with the electromagnetic field, so mechanics were absorbed by electromagnetism. Abraham's electron had a uniform volume/surface charge distribution. Within his theory, he predicted the electron longitudinal (m_L) and transversal (m_T) mass formulas. The mass of a body can be described as the opposition of it being accelerated. Then, to explain the movement of a high-speed electron, it was necessary to define a longitudinal mass, parallel to the direction of motion, and a transversal mass, perpendicular to this direction. Next, Alfred Bucherer (1863-1927) presented his own electron model [6, p. 1134] in 1904, in which the electron was deformable as it moved, but its volume remained constant. He also calculated the corresponding mass components.

On the other side, there was the theory of the electron of Hendrik Lorentz (1853-1928) [5, pp. 67-75], developed between 1892 and 1904. This electron was deformable: it remained as a uniform sphere at rest, but when moving at relativistic speeds, it suffered a contraction in its length in the direction of movement while its transverse dimension didn't change. In the electron's reference system, its own shape remained unaltered. He also calculated the corresponding values of m_L and m_T, which for low velocities were similar to those obtained by Abraham. Lorentz's theory opposed the one of Abraham and his electromagnetic world-picture, which by 1904 was of great importance within the scientific community. But Lorentz needed a new and generalized theory, based in a deformable electron, in order to explain second-order phenomena in the optics of moving bodies in addition to the electron's mass. To do so, he made several assumptions and hypotheses, such as his transformations for space and time. And it is here where the historical line connects with Einstein's theory of relativity of 1905. Starting from

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postulates (all laws of physics are the same in any frame of reference, c is equal from any observer), he got to the same results as Lorentz did. Therefore, since 1906 the last-mentioned electron's theory is also referred as Lorentz-Einstein theory.

Kaufmann, the main character of this project, performed various experiments from 1901 to 1905 analyzing electron's charge-mass changes with velocity. The empirical data obtained proved Lorentz's theory wrong, therefore becoming an evidence against the special theory of relativity. This caused a debate in the physics community of the moment, dividing it between supporters of classical (Abraham's and Bucherer's) and relativistic (Lorentz-Einstein) theories of the electron. Later on, reanalysis of Kaufmann's data was made, as well as subsequent experiments, in order to have a better understanding of the problematic and draw fair conclusions. To get a better perspective of these successes, next chapter is devoted entirely to Kaufmann experiments, made from 1901 to 1905.

III. KAUFMANN EXPERIMENTS (1901-1905)

As it has been said, Kaufmann experiments aimed to observe the variation of the ratio charge-mass (e/m) of the electron. The empirical results established that e/m diminished with the increase of the velocity. Within the framework of classical electrodynamics, this was understood as the increase of the (electromagnetic) mass with the velocity. For his experiments during those years, Kaufmann used the following design [6, p. 1138] (modified it significatively little during those five years): an exterior cylindrical container as a vacuum chamber, with a pair of vertical condenser plates inside. Using radium chloride, he created high-speed beta-rays. These, composed of electrons, went through the section between the condenser plates, which had a horizontal electric field E established between them. This E field gave the electrons a horizontal acceleration, therefore a deviation. After the condenser, the electrons entered a region with a horizontal magnetic field B (parallel to E), which due to the Lorentz force deviated the initial trajectory of the electrons, already changed in the condenser. Figure 1 shows a scheme of this situation: the magnetic field B provokes a deviation of the trajectory in the x-z plane, from the (0,0,0) to the $(x_2, 0, \bar{z})$; adding the effect of the E field, the electron's trajectory is deviated in the x-y plane, so all in all the endpoint in the photographic plate is $(x_2, \bar{y}, \bar{z})$. Concerning the velocity, electrons start with a $\vec{v} = (v_x, 0,0)$, and end up with a $\vec{v} = (v_x, v_y, v_z)$. During all this process, the v_x remained constant. With this construction and some calculations, Kaufmann obtained expressions for $\bar{y}$ and $\bar{z}$. Combining them, Kaufmann got a formula for the charge-mass ratio:

$$\frac{e}{m} = \left[\frac{2Ec^2x_1}{B^2x_2^2(x_2 - x_1)}\right]\frac{\bar{z}^2}{\bar{y}} \qquad (1)$$

Where $x_1$is the vertical coordinate at the superior end of the condenser. Next, the different experiments of Kaufmann are discussed, making special remark to 1905 data.

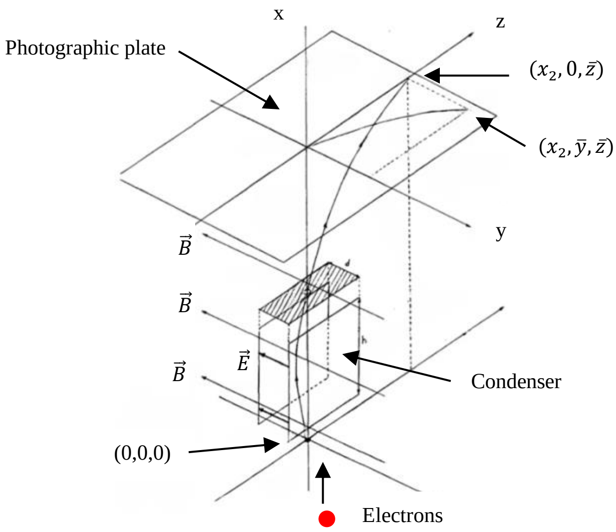

Figure 1. Scheme of Kaufmann's apparatus [6, p. 1139]

A. 1901-1902 experiments

In 1901 Kaufmann published his first experimental results, where he only analyzed the ratio e/m, not comparing any electron theory. He considered the mass had two components: a mechanical/real (M) and an electromagnetic/apparent (m_0) one. Therefore, $m = M + m_0$. For the measures, he considered the apparent mass, which resulted equal to the electromagnetic longitudinal mass (m_L), that later on Abraham would prove it was not appropriate considering a deflection by B field. Kaufmann obtained 5 data points, with the respective $\bar{y}$ and $\bar{z}$ measured. He then calculated e/m, as well as applied the least squares method to the data to obtain the best values of e/M and e/m_0. However, Kaufmann's calculations were not correct because he used the longitudinal mass instead of the transversal, which was the appropriate. Abraham also pointed out that there was only one type of mass, no apparent mass, and it was electromagnetic in its nature. In 1902, Kaufmann repeated the experiments accordingly to Abraham, with the m_T and having modified an analytical error made in the computation of the radius trajectory. Moreover, he took out the E and B fields from the formulas, introducing the apparatus constant k_1 and the constant k_2. Therefore, combining some formulas he got the following simplified equation [6, p. 1141]:

$$\bar{y}\bar{z}^2\psi\left(k_1\frac{\bar{z}}{\bar{y}}\right) = k_2 \qquad (2)$$

Where ψ is a function, different for each theory. From the experimental values of $\bar{y}$ and $\bar{z}$, and the apparatus constant k_1 measured, Kaufmann computed k_2, which was approximately constant in all the measures. Then, he took the mean value of k_2, and used it to compute $\frac{e}{m_0} = 1.84 \cdot 10^7 \frac{emu}{g}$ (units in CGS, $1\ emu = 10\ C$ in SI). This value was quite close to the $1.865 \cdot 10^7$ reference value known by then (calculated by Simon in 1899 using cathodic rays). Then, Kaufmann data confirmed Abraham's theory, as well as that electron's mass was purely electromagnetic.

B. 1905 experiments

In 1903, Kaufmann published more data, but it wasn't until 1905 that he decided to compare the theories of Abraham, Bucherer and Lorentz [2]. The experiment had been improved substantially since 1901, so now data were treated more correctly and precisely, with less approximations. Now the main formula was [6, p. 1142]:

$$\bar{y} = \frac{e}{m_0}\frac{A'}{c^2} f\left(\frac{\bar{z}}{\frac{e}{m_0}\frac{A}{c}}\right) = D' f\left(\frac{\bar{z}}{D}\right) \qquad (3)$$

Where m_0 is the rest mass of the electron, A and A' are constants of the apparatus, and f is a function. Table 1 presents the 9 data points (each of them with different statistical weight) obtained in Kaufmann experiment of 1905, with empirical $\bar{z}$ and $\bar{y}$ in the first two columns. Using the least-squares method with the empirical data, Kaufmann obtained D and D'. Then, with some more calculations, Kaufmann got the $\overline{y_{theo}}$ for each data point. Third, fourth and fifth columns consist of the comparison (using the difference δ) between experimental and theoretical values of $\bar{y}$ for each of the three theories: Abraham (a), Lorentz (b) and Bucherer (c). With these columns, he concluded that it did not allow to decide which of the three models fit the data better.

Kaufmann computed the ratio e/m_0 substituting D and D' for its expressions, obtaining for theories (a), (b) and (c): $\frac{e}{m_0} = 1.823 \cdot 10^7, 1.660 \cdot 10^7, 1.808 \cdot 10^7 \ \frac{emu}{g}$ respectively.

Comparing them with the reference value $1.865 \cdot 10^7$ computed by Simon, Kaufmann affirmed that since Abraham's theory matched his data better than Lorentz's, then Abraham's was the correct one to explain electron properties. However, he concluded that it was not possible to decide between Abraham's and Bucherer's models. In this sense, more experiments had to be made. It is remarkable to mention Kaufmann's conclusions in his own words, in 1905 paper:

"The results above speak against the correctness of Lorentz's, and also consequently of Einstein's, fundamental hypothesis. If one considers this hypothesis as thereby refuted, then the attempt to base the whole of physics, including electrodynamics and optics, upon the principle of relative motion is also a failure." [6, p. 1142]

In 1906, Kaufmann published a complete review of all his previous experiments, where he referred the relativistic model as Lorentz-Einstein theory, as he had analyzed thoroughly Einstein's 1905 article. Kaufmann was the first to mention on print Einstein's special relativity theory, finding many similarities with Lorentz's results and concluding that Einstein's was a generalized and improved version of the latter.

$\bar{z}$	$\overline{y_{exp}}$	$\delta = (\overline{y_{exp}} - \overline{y_{theo}}) \cdot 10^4$		
		(a)	(b)	(c)
0.1350	0.0246	-5	0	-8
0.1919	0.0376	-1	+1	-3
0.2400	0.0502	0	0	0
0.2890	0.0545	-4	-6	-2
0.3359	0.0811	0	-2	+3
0.3832	0.1001	+6	+4	+9
0.4305	0.1205	+4	+3	+5
0.4735	0.1404	-4	-2	-5
0.5252	0.1666	-16	-12	-21

Table 1. Kaufmann's 1905 data and analysis [6, p. 1142]

IV. LORENTZ'S AND PLANCK'S REACTIONS

The 1905 and 1906 data publications of Kaufmann caused a disturbance in the physics community of that moment, and there were different reactions depending on each physicist involved. Good example of this are Lorentz and Max Planck (1858-1947). Lorentz was devastated by the consequences of these experiments, as his following statement expresses: "Unfortunately my hypothesis of the flattening of electrons is in contradiction with Kaufmann's results, and I must abandon it. I am, therefore, at the end of my Latin." [Lorentz to Poincaré, 8 March 1906], [5, p. 334]. Early in 1904, Lorentz had done a complementary analysis of Kaufmann's 1902 data, using an analogous proceeding but with his own theoretical model. Lorentz concluded that either theory had a good fit for the data.

Planck's reaction was completely different: he defended that Lorentz-Einstein's model was correct, and that the conclusions reached by Kaufmann were due to an error in the experiments of the results' analysis. That is why, in 1906, he reanalyzed Kaufmann's experiments of 1905 with his own calculations. Before that, Planck had presented an important article [3], where he modified Newton's second law for relativistic electrons moving in an electromagnetic field (first obtained by Einstein in 1905, although he did some errors in the formulation). In fact, Planck was the first to obtain this kind of dynamical equations involving relativistic forces:

$$\frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\beta^2}}\right) = e\left(E + \frac{v}{c} \times B\right) \qquad (4)$$

Then, he derived the electron's motion equations for Lorentz's and Abraham's theories, calculating $\overline{y_{theo}}$ from the nine experimental $\bar{z}$ values of Kaufmann, and extracting $\beta = v/c$ too. In his first calculations, Planck admitted that Abraham's theory provided a better fit than Lorentz-Einstein's did. However, he found one data point with $\beta > 1$, which disconcerted him quite a lot. Then, he wasn't satisfied with these conclusions, so in 1907 published a paper focusing on other errors that Kaufmann may have made in the experiments. In particular, concerning the quality of the vacuum made by Kaufmann. If it wasn't maintained enough, the beta-rays could ionize residual air molecules, consequently reducing the E strength between the condenser plates. So, this time, Planck recalculated Kaufmann's data, now using a free parameter α in the expression of the E field. Since this should be a characteristic value of the apparatus, α should be the same for all the data. Then, Planck compared the variation of this parameter in Abraham's theory (5%) and in Lorentz-Einstein's model (2%). Under this assumption, Planck concluded that Kaufmann's data was a better fit to Lorentz-Einstein theory. It should be noted that even this assumption is not very solid, Planck's important contribution is that he was the first to prove that Kaufmann experiments didn't present a real threat to special theory of relativity.

V. 1906 CONGRESS OF GERMAN NATURAL SCIENTISTS IN STUTTGART

This important congress, involving some of the most world-renowned physicists at that moment, was held on September 19th, 1906. It's remarkable the discussion they had about Kaufmann's 1905 experiments and its consequences for special relativity. Planck started the discussion presenting his previous calculations of $(\bar{y}, \bar{z})$ from his Lagrangian formalism, using $\bar{z}$ from Kaufmann's data. He found that Abraham's model $\bar{y}$ were closer to the observed corresponding values. However, Planck exposed that a comparison of the calculated and empirical values of $\bar{y}$ "is in my opinion not a definitive verification of [Abraham's theory] and a refutation [of Lorentz-Einstein's]." [5, pp. 233]. Moreover, he presented his deduced value of $\beta > 1$, obtained applying his calculations to one of Kaufmann's empirical data points. He used it as an example that Kaufmann's theoretical interpretation of measured quantities was unclear.

To these statements, Kaufmann responded presenting his "reduced curve" containing the empirical and calculated data. There, Lorentz-Einstein's theory calculated data deviated 10-12% from empirical data, meanwhile in Abraham's model this deviation was 3-5%, substantially smaller. In Kaufmann's opinion, this was a sufficient argument towards the correctness of Abraham's theory.

Planck replied that unknown errors could conspire to bring Lorentz-Einstein's theory into better agreement with the empirical data. Thus, Planck concluded that "from these bare data [i.e. $(\bar{y}, \bar{z})$] the fact that the deviation on one theory is smaller would not follow a preference for it." [5, p. 233]

After that observation, Bucherer entered the discussion, stating that his theory of the electron was not sufficiently developed to be analyzed in subsequent experiments, including Planck's formulations. From then on, only Abraham's and Lorentz-Einstein's models would be compared.

Abraham also took part in the discussion, starting with the following comment: "When you look at the numbers you conclude from them that the deviations of the Lorentz-Einstein's theory are at least twice as big as those of mine, so you may say that the sphere theory [his theory] represents the deflection of beta-rays twice as well as the *Relativitätstheorie* [Lorentz-Einstein's model]." [5, p. 234]. He continued arguing the substantial difference between the two theories: the fact that Lorentz-Einstein's theory didn't consider an electromagnetic world-picture, because of considering a deformable electron. To this statement, Planck replied that both theories were based on postulates, which were logically incompatible; he then remarked that was clearly in favor of Lorentz-Einstein, although more experiments had to be done to confirm its correctness. Due to the extreme difficulty of Kaufmann's experiments, there might be unknown errors influencing the results still having to be discovered.

VI. EINSTEIN'S REACTION

As it has been stated, there were different reactions to Kaufmann's 1905 experiments in the special relativity community of supporters: Lorentz was devastated and gave up on his own theory, while Planck was affected and willing to prove them wrong. However, Einstein remained unaltered by Kaufmann's conclusions regarding his theory, he wasn't concerned at all. In fact, Einstein ignored Kaufmann's results until 1907, when in an article [4] he presented Kaufmann's empirical "reduced curve". There, $\bar{y}$ and $\bar{z}$ data from Kaufmann are compared to those of relativity theory [5, pp. 344]. Einstein's opinion about Kaufmann data is perfectly summarized in his statement:

"The theories of the electron's motion of Abraham and Bucherer [agree better with Kaufmann's data] than the relativity theory. In my opinion both theories have a rather small probability, because their fundamental assumptions concerning the mass of moving electrons are not explainable in terms of theoretical systems which embrace a greater complex of phenomena." [5, p. 341]

To sum up, Einstein congratulated Kaufmann for his accurate experimental analysis and comparison, accepting a better fit for the classical theories (Abraham's and Bucherer's), but considered these two incorrect because of their complicated expressions for the electron's mass. As Planck had asserted, Einstein considered that further data were necessary to draw a sufficient strong conclusion.

VII. BESTELMEYER, BUCHERER AND SUBSEQUENT EXPERIMENTS

In 1907, after Planck's reanalysis and Einstein's discussion of Kaufmann's data, it was still not clear which theory (Abraham's or Lorentz-Einstein's) fit the data best, and therefore was correct to explain the electron's behavior. More experiments had to be done. That same year, Adolf Bestelmeyer (1875-1957) had used cathode rays to determine the ratio e/m_0 for each theory, considering small β. However, it was not possible to decide between any of the two electron models with the results obtained.

It was Bucherer, who in 1908 repeated Kaufmann's experiments (more precise and improved) using beta-rays, that obtained more conclusive results. He listed the different e/m_0 obtained for different β, for each theory. The criterion he used was the following (and that next experiments would also apply): the more constant the values of e/m_0 as β varied, the more correct the respective theory was. Bucherer concluded that Lorentz's values of the ratio were more constant, and therefore Lorentz's theory was to be preferred between the two. In a letter of September 7th, 1908, Bucherer addressed Einstein with the following sentence: "…by means of careful experiments, I have elevated the validity of the principle of relativity beyond any doubt" [5, p. 349]. However, some physicists questioned Bucherer's data because of fringing effects on his condenser plates.

Finally, in 1914 Neumann confirmed Bucherer's conclusions, using a refinement of his method with a similar apparatus, and obtaining a more constant e/m_0 in Lorentz's theory for 26 data points for $\beta \in [0.31952, 0.80730]$. C.Guye and C. Lavanchy confirmed again, in their experiments of 1915, that Lorentz's theory was the correct one concerning the electron model.

VIII. CONCLUSIONS

Kaufmann experiments are a good example that science does not operate in a simple refutational way, that is, with the rejection of hypothesis and theories only because a single experiment contradicts them. Even though Kaufmann's data were a better fit to Abraham's and Bucherer's theories, Planck didn't reject relativity theory even after verifying Kaufmann's calculations. And the same happened with Einstein, who didn't intervene in the discussion until some more experiments and reanalysis (Planck's and Bestelmeyer's) had been done, questioning the veracity of Kaufmann's results. It can be induced that Einstein had the intuition that Kaufmann was wrong on his experiments and conclusions. And this intuition was right, because after Bucherer's (1908) and Neumann's (1914) experiments, it became clear that Lorentz-Einstein's theory was the one that explained better the behavior of high-speed electrons. What initially seemed to be a strong argument against special relativity, ended up as a non-influential obstacle in the proper development of the latter.

By the time that Neumann's results appeared, special relativity was already established as the predominant theory in physics, giving rise to many other scientific breakthroughs. On the downside, Abraham theory was proved wrong, and with it followed the electromagnetic world-picture. Special relativity was too important and general to be proved wrong due to some punctual experiments.

In fact, the validity of special relativity was also confirmed by other experiments/results during that time. A remarkable one is Arnold Sommerfield's 1916 introduction of relativity in the calculations of the atomic model, obtaining new spectral lines for the Hydrogen. He derived the fine-structure constant, which was an impressive achievement both in relativity and quantum. [7, p. 260]

The theory of Einstein's general relativity (1915), and its experimental verification in 1919, through the observation (leaded by Arthur Eddington) of the deviation of the light emitted by stars during a solar eclipse (due to the curvature of light when passing near a big mass; that is, the sun), were more key results to end reticence's over special relativity among the scientific community. [8, pp. 184-185]

Different reactions have been studied from the physicists involved in the abovementioned debate: Kaufmann's conviction, Abraham's approval, Lorentz's desperation, Planck's resilience, Einstein's indifference, etc. All of them took part at some point in this brief but intense discussion over relativity theory and its acceptance after Kaufmann's results. From them, it can be subtracted that contrast and recalculation of experiments is vital when it comes to drawing important conclusions. It is vital for the good development of science and its theories to construct an international scientific community and network, as there was in Stuttgart in that congress of 1906. It gives the necessary diversity, contrast of opinions and points of view to solve a particular problematic.

This article has intended to mix the technical approach of Cushing [6] with the narrative approach of Miller [5], along with the main primary sources, resulting in a balanced description of Kaufmann's experiments and its consequent reanalysis and discussions, overall giving a new perspective to these historical events. Finally, it would be interesting to give continuity to this article with the study of subsequent experiments related to Kaufmann's results, and that corroborated Lorentz-Einstein's theory in front of Abraham's.

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